EE 2030 Linear Algebra Chi-chao Chao
Spring 2011

Homework Assignment No. 1
Due 10:10am, March 11, 2011

`Reading:` Strang, Chapters 1 and 2.

`Problems for Solution:`

1. Find the pivots and the solutions for both systems of linear equations:

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & -1 & -3 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 0 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 \\ 2 & 4 & 1 & -2 \\ 3 & 1 & -2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ -1 \\ 3 \end{bmatrix}.$$

2. Find elimination matrices $\boldsymbol{E}_{21}$ then $\boldsymbol{E}_{32}$ then $\boldsymbol{E}_{43}$ to change $\boldsymbol{K}$ into $\boldsymbol{U}$:

$$\boldsymbol{E}_{43}\boldsymbol{E}_{32}\boldsymbol{E}_{21} \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 3/2 & -1 & 0 \\ 0 & 0 & 4/3 & -1 \\ 0 & 0 & 0 & 5/4 \end{bmatrix}.$$

Apply those three steps to the identity matrix $\boldsymbol{I}$, to obtain the result of multiplying $\boldsymbol{E}_{43}\boldsymbol{E}_{32}\boldsymbol{E}_{21}$.

3. For each of the matrices

$$\boldsymbol{A} = \begin{bmatrix} 1 & -2 & 1 \\ 2 & -1 & -1 \\ -2 & -5 & 7 \end{bmatrix} \quad \text{and} \quad \boldsymbol{B} = \begin{bmatrix} 1 & 1 & 0 & -1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 0 & -3 \\ -1 & -1 & 1 & 1 \end{bmatrix}$$

determine whether the matrix is invertible. If so, find its inverse.

4. Compute $\boldsymbol{L}$ and $\boldsymbol{U}$ for this matrix:

$$\boldsymbol{A} = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}.$$

Find four conditions on a, b, c, d to get $\boldsymbol{A} = \boldsymbol{L}\boldsymbol{U}$ with four pivots.

5. Factor the following symmetric matrices into $\boldsymbol{A} = \boldsymbol{L}\boldsymbol{D}\boldsymbol{L}^T$:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 3 & 5 \\ 3 & 12 & 18 \\ 5 & 18 & 30 \end{bmatrix} \quad \text{and} \quad \boldsymbol{A} = \begin{bmatrix} a & b \\ b & d \end{bmatrix}.$$

6. Prove the following statements:

   (a) A lower (upper) triangular matrix with unit diagonal is invertible and its inverse is still lower (upper) triangular with unit diagonal.

   (b) The product of two lower (upper) triangular matrices with unit diagonal is still lower (upper) triangular with unit diagonal

   (c) The product of a lower (upper) triangular matrix and a diagonal matrix is lower (upper) triangular.

7. If $\boldsymbol{A} = \boldsymbol{L}_1\boldsymbol{D}_1\boldsymbol{U}_1$ and $\boldsymbol{A} = \boldsymbol{L}_2\boldsymbol{D}_2\boldsymbol{U}_2$, where the $\boldsymbol{L}$'s are lower triangular with unit diagonal, the $\boldsymbol{U}$'s are upper triangular with unit diagonal, and $\boldsymbol{D}$'s are diagonal matrices with no zeros on the diagonal, prove that $\boldsymbol{L}_1 = \boldsymbol{L}_2$, $\boldsymbol{D}_1 = \boldsymbol{D}_2$, and $\boldsymbol{U}_1 = \boldsymbol{U}_2$. (*Hint:* The proof can be decomposed into the following two steps:

   (a) Derive the equation $\boldsymbol{L}_1^{-1}\boldsymbol{L}_2\boldsymbol{D}_2 = \boldsymbol{D}_1\boldsymbol{U}_1\boldsymbol{U}_2^{-1}$ and explain why one side is lower triangular and the other side is upper triangular.

   (b) Compare the main diagonals in the equation in (a), and then compare the off-diagonals.)

8. Factor the following matrix into $\boldsymbol{PA} = \boldsymbol{LU}$. Also factor it into $\boldsymbol{A} = \boldsymbol{L}_1\boldsymbol{P}_1\boldsymbol{U}_1$.

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 3 & 8 \\ 2 & 1 & 1 \end{bmatrix}.$$